TEMPERATURE DISTRIBUTION IN THE CYLINDER BLOCK OF AN AXIAL PISTON PUMP

V. L. Rvachev, A. P. Slesarenko, and N. D. Syzova

UDC 536.2.01

A calculation is presented on the three-dimensional temperature distribution in the cylinder block of an axial piston pump; the heat-conduction problem is solved by the joint use of finite integral transformations, the structural method, and the Bubnov–Galerkin method. The results are compared with experiment.

An axial piston pump consists of a cylindrical block with holes passing through the height of the cylinder; Fig. 1 is a simplified scheme for a symmetrical element in such a pump. The characteristic parameters in an element are the radii r_1, r_2, r_3, the height of the cylindrial block d, and the angle φ_1 characterizing the symmetrical element. Pumps of this type are widely used in various building machines and road vehicles, aviation engines, etc.

The cylinder block is heated by the friction arising along the contact surfaces with the components moving relative to the block, mainly the piston. The surfaces S_1, S_2, S_3 provide heat transfer from the block into the environment in oil. At S_4, heat is produced by friction with the mating part. The surfaces S_5 and S_6 are insulated. Within the block, S_7 supplies a constant heat flux q_7.

The temperature distribution in the symmetrical element is determined by solving the following heat-conduction problem:

$$\Delta T = -F, \tag{1}$$

$$\left(\frac{\partial T}{\partial z} + \mathrm{Bi}_2 T\right)\Big|_{S_2} = f_2; \quad \left(\frac{\partial T}{\partial z} - \mathrm{Bi}_4 T\right)\Big|_{S_4} = f_4,$$

$$\left(-\frac{\partial T}{\partial \rho} + \mathrm{Bi}_1 T\right)\Big|_{S_1} = f_1; \quad \left(\frac{\partial T}{\partial \rho} + \mathrm{Bi}_3 T\right)\Big|_{S_3} = f_3, \quad \frac{\partial T}{\partial \nu_5}\Big|_{S_5} = \frac{\partial T}{\partial \nu_6}\Big|_{S_6} = 0;$$

$$\frac{\partial T}{\partial \nu_7}\Big|_{S_7} = q_7, \tag{3}$$

where $\nu_k (k = 5, 6, 7)$ is the direction of the exterior normal to S_k, $\mathrm{Bi}_k = 13.3$ $(k = 1, 2, 3, 4)$; $F = F_1 L^2 = 3.48$ deg; $f_k = \mathrm{Bi}_k T_{am} = 658.1$ deg $(k = 1, 2, 3, 4)$; $q_7 = q_7^* L = -0.95$ deg; $L = 1$ m; $z = z_1 L^{-1}$; $x = x_1 L^{-1}$; $y = y_1 L^{-1}$; $d_1 = dL^{-1}$; $\rho = \sqrt{x^2 + y^2}$.

We apply a finite integral transformation to (1) and boundary conditions of (3)

$$\bar{T}(x, y, \gamma) = \int_0^{d_1} T(x, y, z) K(z, \gamma)\, dz, \tag{4}$$

where the kernel is

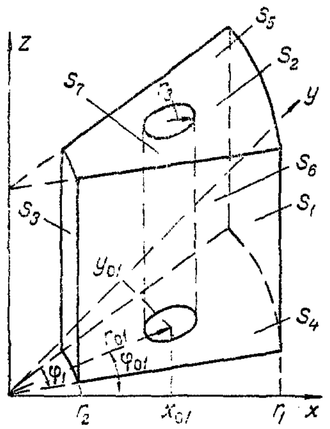
$$K(z, \gamma) = \cos\gamma z + \frac{\mathrm{Bi}_4}{\gamma} \sin\gamma z \tag{5}$$

and is a nonzero solution to the Sturm–Liouville problem for the equation

$$\frac{d^2K}{dz^2} + \gamma^2 K = 0, \tag{6}$$

Engineering Problems Institute, Academy of Sciences of the Ukrainian SSR, Kharkov.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 39, No. 3, pp. 527-531, September, 1980. Original article submitted June 1, 1979.

0022-0841/80/3903-1025$07.50



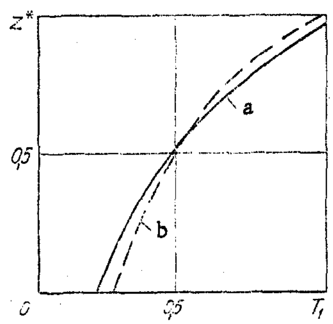


Fig. 1 Fig. 2

Fig. 1. Scheme for an axisymmetrical element of a cylinder block in an axial piston pump.

Fig. 2. Temperature distribution at the surface of a cylinder block: a) from proposed calculation; b) from experiment ($z^* = z_1 d^{-1}$; $T_1 = T\,T_{am}^{-1}$).

$$\left(\frac{dK}{dz} + \mathrm{Bi}_2 K\right)\Big|_{z=d_1} = 0;\quad \left(\frac{dK}{dz} - \mathrm{Bi}_4 K\right)\Big|_{z=0} = 0, \tag{7}$$

and $\gamma = \gamma_m$ ($m = 1, 2, \ldots$) are the roots of the transcendental equation

$$\mathrm{tg}\,\gamma_m d_1 = \frac{(\mathrm{Bi}_2 + \mathrm{Bi}_4)\,\gamma_m}{\gamma_m^2 - \mathrm{Bi}_2\mathrm{Bi}_4}.$$

We obtain the following boundary-value problem in the transform region:

$$\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2} - \gamma^2 \bar{T} = -F + f_4 K(0,\ \gamma) - f_2 K(d_1, \gamma) = \bar{F}_1, \tag{8}$$

$$\left(-\frac{\partial \bar{T}}{\partial \rho} + \mathrm{Bi}_1 \bar{T}\right)\Big|_{S_1} = \bar{f}_1;\quad \left(\frac{\partial \bar{T}}{\partial \rho} + \mathrm{Bi}_3 \bar{T}\right)\Big|_{S_3} = \bar{f}_3,$$

$$\frac{\partial \bar{T}}{\partial \nu_5}\Big|_{S_5} = \frac{\partial \bar{T}}{\partial \nu_6}\Big|_{S_6} = 0;\quad \frac{\partial \bar{T}}{\partial \nu_7}\Big|_{S_7} = \bar{q}_7, \tag{9}$$

where $\bar{f}_i = \int_0^{d_1} f_i K(z,\ \gamma)\,dz$ $(i = 1,3)$; $\bar{q}_7 = \int_0^{d_1} q_7 K(z,\ \gamma)\,dz$; the structure of the solution to (8) and (9) that satisfies the conditions of (9) precisely is represented [1-5] as

$$\bar{T}(x,\ y,\ \gamma) = \bar{\Phi}_0(x,\ y,\ \gamma) + \sum_{i,j} C_{ij}(\gamma)\,X_{ij}(x,\ y), \tag{10}$$

where

$$\bar{\Phi}_0 = -\omega\,[\omega_3\omega_5\omega_6\omega_7\bar{f}_1 + \omega_1\omega_5\omega_6\omega_7\bar{f}_3 + \omega_1\omega_3\omega_5\omega_6\bar{q}_7]\,\beta^{-1};$$

$$X_{ij} = \Phi_{ij} + \omega\Phi_{ij}\delta\beta^{-1} - \omega D_1\Phi_{ij};\ \delta = \mathrm{Bi}_1\omega_3\omega_5\omega_6\omega_7 + \mathrm{Bi}_3\omega_1\omega_5\omega_6\omega_7;$$

$$\beta = \omega_3\omega_5\omega_6\omega_7 + \omega_1\omega_5\omega_6\omega_7 + \omega_1\omega_3\omega_6\omega_7 + \omega_1\omega_3\omega_5\omega_7 + \omega_1\omega_3\omega_5\omega_6;$$

$$\omega_1 = (x^2 + y^2 - r_{11}^2)(2r_{11})^{-1};\ \omega_3 = (r_{22}^2 - x^2 - y^2)(2r_{22})^{-1};$$

$$\omega_5 = (k_1 x - y)(k_1^2 + 1)^{-1};\ \omega_6 = (y - k_2 x)(k_2^2 + 1)^{-1};$$

$$\omega_7 = [(x - x_{01})^2 + (y - y_{01})^2 - r_{33}^2](2r_{33})^{-1};$$

$$\omega = [\omega_1\omega_3(r_{22}^2 - r_{11}^2)\Lambda_0\omega_7]\Lambda_0(\omega_5\Lambda_0\omega_6);\ \eta_1\Lambda_0\eta_2$$

$$= \eta_1 + \eta_2 - \sqrt{\eta_1^2 + \eta_2^2};$$

$$\Phi_{ij} = \rho^{\frac{2\pi}{\varphi_2}i}\cos\frac{2\pi}{\varphi_2}j\varphi;\ \rho = \sqrt{x^2 + y^2},\ \varphi = \mathrm{arctg}\frac{y}{x};\ r_{ii} = r_i L^{-1}.$$

TABLE 1. Results from Temperature Calculation

x	y				
	0,2	0,4	0,6	0,8	1,0
0,2	0,055000 0,054628	0,075000 0,074458	0,108333 0,106092	0,155000 0,153752	0,215000 0,213999
0,4	0,075000 0,074458	0,095000 0,093144	0,128333 0,127786	0,175000 0,174052	0,235000 0,234119
0,6	0,108333 0,106092	0,128333 0,127786	0,161333 0,159998	0,208333 0,207428	
0,8	0,155000 0,153752	0,175000 0,174052			
1,0	0,215000 0,213999				

In the calculation we envisage a symmetrical element of the cylinder block for $r_1 = 0.026$ m; $r_2 = 0.004$ m; $r_3 = 0.008$ m; $d = 0.048$ m; $k_1 = 4$; $k_2 = 0.25$; $\varphi_2 = 1.0804...$; $x^*_{01} = 0.0812$ m; $y^*_{01} = 0.0915$ m.

The Bubnov—Galerkin system is as follows for the undefined coefficients $C_{ij}(\gamma)$ in (10):

$$\sum_{i+j=0}^{n} (A_{ijks} + B_{ijks}\gamma^2) C_{ij}(\gamma) = E_{ks}(\gamma) , \tag{11}$$

where $k+s=0, 1,...,n$;

$$A_{ijks} = \iint_{\Omega_1} \Delta X_{ij} X_{ks} d\Omega_1; \quad B_{ijks} = -\iint_{\Omega_1} X_{ij} X_{ks} d\Omega_1;$$

$$E_{ks}(\gamma) = \iint_{\Omega_1} [-\overline{F} + f_4 K(0, \gamma) - f_2 K(d, \gamma) - \Delta\overline{\Phi}_0 + \gamma^2 \overline{\Phi}_0] X_{ks} d\Omega_1,$$

and Ω_1 is the cross section of the body.

The solution to (11) is found in the form

$$C_{ij}(\gamma) = \left[\sum_{k+s=0}^{n} E_{ks}(\gamma) \Delta_{ksij}(\gamma) \right] [\Delta(\gamma)]^{-1} , \tag{12}$$

where $i+j=0, 1,...,n$;

$$\Delta(\gamma) = \begin{vmatrix} A_{0000} + B_{0000}\gamma^2 & \ldots & A_{0n00} + B_{0n00}\gamma^2 \\ \cdot & \cdot\cdot\cdot & \cdot \\ A_{000n} + B_{000n}\gamma^2 & \ldots & A_{0n0n} + B_{0n0n}\gamma^2 \end{vmatrix};$$

where $\Delta_{ksij}(\gamma)$ are the algebraic complements of the elements $(A_{ijks} + B_{ijks}\gamma^2)$ in the determinant $\Delta(\gamma)$.

The following is the formula for reversing the finite integral transformation:

$$T(x, y, z) = \sum_{m=1}^{p} \overline{T}(x, y, \gamma) \left[\int_0^{d_1} K^2(z, \gamma_m) dz \right]^{-1} K(z, \gamma_m) \tag{13}$$

for $p \to \infty$.

The function T(x, y, z) is defined by this series and is a solution to the boundary-problem of (1)-(3) in any case where the series allows of termwise differentiation twice with respect to x, y, and z.

Figure 2 gives the temperature distribution at the surface of the cylinder block together with measurements (broken line) made at the All-Union Road-Building Machinery Research Institute in Moscow. The calculation from (13) was performed for 15 coordinate functions and $p = 50$.

As a test example, we consider the case where the determination of the temperature distribution in the symmetrical element (Fig. 1) amounts to solution of the following boundary-value problem:

$$\Delta T = 1\,, \tag{14}$$

$$\left.\frac{\partial T}{\partial \rho}\right|_{S_1} = \frac{r_1}{3}\,; \quad \left.\frac{\partial T}{\partial \rho}\right|_{S_3} = \frac{r_2}{3}\,; \quad \left.\frac{\partial T}{\partial \nu_7}\right|_{S_7} = f_7\,; \tag{15}$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=0} = 0; \quad \left.\frac{\partial T}{\partial z}\right|_{z=d_1} = \frac{d_1}{3}\,; \tag{16}$$

$$\left.\frac{\partial T}{\partial \nu_5}\right|_{S_5} = \left.\frac{\partial T}{\partial \nu_6}\right|_{S_6} = 0\,, \tag{17}$$

where

$$f_7 = \frac{1}{3r_3}\,[r_3^2 + \rho r_{01}\cos(\varphi - \varphi_{01}) - r_{01}^2]\,.$$

The integral transformation of (4) is applied to (14) and to (15) and (17) with the transformation kernel $K(z, \gamma) = \cos \gamma z = \cos \frac{m\pi}{d_1} z$ to represent the structure of the solution in the transform region in the form of (10) where $i = 1, 2$:

$$\bar{f}_i = \frac{r_i}{3}\int_0^{d_1} K(\gamma, z)\,dz; \quad \bar{f}_7 = \bar{q}_7 = \int_0^{d_1} f_7 K(\gamma, z)\,dz\,.$$

In the calculation we envisage a symmetrical element for the cylinder block for $r_{11} = 0.1$; $r_{22} = 1.0$; $r_{01} = 0.5$; $\varphi_{01} = 2\pi/7$.

The solution to (14)-(17) is derived from (13).

Table 1 gives the calculated temperature distribution for $z = 0.5$ with use of the exact solution

$$T = \frac{x^2 + y^2 + z^2}{6}$$

and the result from (13) for 15 coordinate functions and $p = 50$.

NOTATION

T, temperature of element; λ, thermal conductivity; Bi, Biot number; T_{am}, ambient temperature; x_1, y_1, z_1, coordinates; $F_1 = W\lambda^{-1}$; W, specific power of sources.

LITERATURE CITED

1. C. J. Tranter, Integral Transforms in Mathematical Physics, Halsted Press (1972).
2. G. N. Polozhii, Equations of Mathematical Physics [in Russian], Vysshaya Shkola, Moscow (1964).
3. V. L. Rvachev and A. P. Slesarenko, Logic Algebra and Integral Transformations in Boundary-Value Problems [in Russian], Naukova Dumka, Kiev (1976).
4. V. L. Rvachev and A. P. Slesarenko, Logic-Algebra and Projection Methods in Heat-Transfer Problems [in Russian], Naukova Dumka, Kiev (1978).
5. V. L. Rvachev, A. P. Slesarenko, and N. D. Syzova, "Calculationof the three-dimensional steady-state temperature distribution within a straight prism," Dop. Akad. Nauk Ukr. SSR, Ser. A, No. 10 (1975).